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33. Proposed by C. E. WHITE, Trafalgar, Indiana.

Show that every algebraic equation of the nth degree, n being greater than two, which is complete in its terms may be transformed into an infinite number of equations which want their second term.

Solution by the PROPOSER.

It is shown in treatises on Higher Algebra that an equation may be changed into another equation of the same degree, but which wants its second term. Now let $x^n+mx^{n-2}+nx^{n-3}+\ldots\ldots+px^2+qx+r=0$ represent this first derived equation, and let

$(x^{n-1}+ax^{n-2}+bx^{n-3}+cx^{n-4}+\ldots\ldots+fx^2+gx+h)(x-a)=0$ represent it as factored. Equating coefficients, we have

$$\begin{aligned}
m&=-a^2+b\ldots.(1)\\
n&=-ab+c\ldots.(2)\\
&\ldots\ldots\ldots\ldots\ldots\ldots\\
p&=-af+g\ldots.(n-3)\\
q&=-ag+h\ldots.(n-2)\\
r&=-ah\ldots.(n-1).
\end{aligned}$$

Now it is easily seen that we can derive without difficulty an equation in h by eliminating a from $(n-1)$ and g from $(n-2)$ and f from $(n-3)$, etc. Moreover, since $h=-\dfrac{r}{a}$, the derived equation in h will contain its second term.

Now, by the method by which we derived the first equation, we may derive from the equation in h another equation of the same degree wanting its second term. From this second derived equation we may derive a third, etc. Hence by continuing the process it is possible to derive an infinite number of such equations. Moreover, the first derived equation may be considered as derived from preceding equations; hence we may find an infinite number of preceding equations of the same form. By every transformation we change the value of the constant term; hence, it is possible that it may take the value zero in one of the derived equations, thus enabling us to find, at least, one root of the equation. Consider the cubic equation, and let x^3+qx+r represent its first derived equation, then, $x^3-\frac{1}{3}q^2x-\frac{2}{27}q^3-\frac{1}{4}r^2=0$ will represent its second derived equation Now, if q be negative and $\frac{2}{27}q^3=\frac{1}{4}r^2$, as in the particular case $x^3-6x+4=0$, the second derived equation reduces to the form $x^3-\frac{1}{3}q^2x=0$. Whence, $x=0,\ \pm\dfrac{1}{\sqrt{3}}q$, from which we easily find the roots of the first derived to be $x=-\dfrac{3r}{q},\ \dfrac{3(1+\sqrt{3})r}{2q}$, and $\dfrac{3(1-\sqrt{3})r}{2q}$

It should be observed that all cubic equations that can be so resolved belong to the irreducible case.

Also solved by *H. C. Whitaker*.